Short Paper: Strategic Contention Resolution in Multiple Channels with Limited Feedback

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Abstract. We consider a game-theoretic setting of contention in communication networks. In a *contention game* each of $n \geq 2$ identical players has a single information packet that she wants to transmit in a fast and selfish way through one of $k \geq 1$ multiple-access channels by choosing a protocol. Here, we extend the model and results of the single-channel case studied in [2] by providing equilibria characterizations for more than one channels, and giving specific anonymous, equilibrium protocols with finite and infinite expected latency. For our equilibrium protocols with infinite expected latency, all players, with high probability transmit successfully in optimal time, i.e. $\Theta(n/k)$.

Keywords: Contention resolution, Multiple channels, Acknowledgement-based protocol, Ternary feedback, Game theory

1 Introduction

The need for multiple channels in communications has become clear in today's technologies. Robustness and high throughput are two main goals that multiple-channels communication systems try to achieve, since dependence from a small group of nodes in a network as well as collision of packets that are transmitted on the same node are the issues from which single-channel broadcast communications suffer. Many works in the Electrical and Electronics Engineering community have so far considered *multi-channel* medium access control (MAC) protocols (e.g. [6]) which have been shown to achieve higher throughput and lower delay than the single-channel MAC protocols. The limited feedback in such systems is caused by the *multi-channel hidden terminal problem* ([7]). To the authors' knowledge, *strategic* behaviour in such multi-channel systems is limited to the Aloha protocol ([5]), contrary to the case of single-channel systems (e.g. [1]).

For equilibrium protocols, a desired property is *anonymity*, that is, protocols which do not use player IDs. If a players' protocol depended on her ID, then equilibria are simple, but can be unfair as well; scheduling each player's transmission through a priority queue according to her ID is an equilibrium. The only works on acknowledgement-based, equilibrium protocols, is by Christodoulou et

* The work of this author was partially supported by the ERC Project ALGAME.

al. [2,3] which consider only a single channel. Among other results, they give the unique, anonymous, equilibrium protocol with finite expected latency for 2 players, and an efficient protocol with infinite expected latency for at least 3 players. The existence of a symmetric equilibrium with finite expected latency remains an open problem, even for three players. However, for the settings with 2 and 3 transmission channels, we manage to present simple, anonymous protocols for up to 4 and 5 players respectively.

In this short paper, we examine the problem of *strategic contention resolution* in multi-channel systems, where obedience to a suggested protocol is not required. We provide two types of equilibrium protocols. The first type (Sect. 3) describes an anonymous, equilibrium protocol that yields finite expected time of successful transmission to a player. Similarly, the second type (Sect. 4) describes an anonymous, equilibrium protocol which yields infinite expected latency to a player but is also *efficient*, that is, all players transmit successfully within $\Theta(\frac{\#players}{\#channels})$ time with high probability. The latter result makes clear the advantage (with respect to time efficiency) that multiple channels bring to a system with strategic users, which is that the time until all players transmit successfully with high probability is inversely proportional to the number of available channels.

2 The Model and Definitions

Game structure. We define a *contention game* as follows. Assume a set of players $[n] = \{1, 2, \dots, n\}$ and a set of channels $K = \{1, 2, \dots, k\}$. Each player has a single packet that needs to be sent through a channel in K, without caring about the identity of the channel. All players know n and K. Time is discrete, i.e. $t = 1, 2, \dots$. The players that have not yet successfully transmitted their packet are called *pending* and initially all n players are pending. At any t, a pending player i has a set $A = \{0, 1, 2, \dots, k\}$ of *pure strategies*: a pure strategy $a \in A$ is the action of choosing channel $a \in K$ to transmit her packet on, or no transmission ($a = 0$). At time t, a *(mixed) strategy* of a player i is a probability distribution over A that potentially depends on information that i has gained from the process based on previous transmission attempts. If exactly one player transmits on a channel in a given slot t, then her transmission is *successful*, she is no longer pending, and the game continues with the rest of the players. However, whenever two or more players try to transmit on the same channel at the same time slot, a collision occurs and they remain pending. The game continues until there are no pending players.

Transmission protocols. Let $X_{i,t} \in A$ be the channel-indicator variable that keeps track of the identity of the channel where player i attempted transmission at time t; value 0 indicates no transmission attempt. An *acknowledgement-based* protocol uses very limited channel feedback. After each time step t, the information received by a player i who transmitted during t is whether her transmission was successful (in which case she gets an acknowledgement and exits the game) or whether there was a collision. Let $\overrightarrow{h}_{i,t}$ be the vector of the *personal transmission history* of player i up to time t, i.e. $\overrightarrow{h}_{i,t} = (X_{i,1}, X_{i,2}, \ldots, X_{i,t})$. A *decision rule* $f_{i,t}$ for a pending player i at time t, is a function that maps $\overrightarrow{h}_{i,t-1}$ to a strategy, with elements $\Pr(X_{i,t} = a|\overrightarrow{h}_{i,t-1})$ for all $a \in A$. For a player $i \in N$, a *(transmission) protocol* f_i is a sequence of decision rules $f_i = \{f_{i,t}\}_{t\geq 1} = f_{i,1}, f_{i,2}, \ldots$. When the context is clear enough we will drop some of the indices accordingly.

Individual utility and equilibria. For a *protocol profile* $\overrightarrow{f} = (f_1, f_2, \ldots, f_n)$, we denote the *expected latency* of player $i \in [n]$, given a history $\overrightarrow{h}_{i,t}$ by $C_i^{\overrightarrow{f}}(\overrightarrow{h}_{i,t})$. We say that $\overrightarrow{f}$ is an *equilibrium* if for any transmission history $\overrightarrow{h}_t$ the players cannot decrease their expected latency by unilaterally deviating after t.

3 Equilibria with Expected Latency $< \infty$

Nash equilibria characterization. Here we provide a characterization of general equilibria (both symmetric and asymmetric) for an arbitrary number of channels $k \geq 1$ and players $n \geq 2$.

Let $\overrightarrow{f} = (f_1, f_2, \ldots, f_n)$ be a tuple of acknowledgement-based protocols (not necessarily anonymous) for the n players. For a (finite) positive integer τ^*, and a given history $h_{i,\tau^*} = (a_{i,1}, a_{i,2}, \ldots, a_{i,\tau^*})$, define for player i the protocol

$$g_i = g_i(h_{i,\tau^*}) \triangleq \begin{cases} (\Pr\{X_{i,t} = a_{i,t}\} = 1) & , \text{for } 1 \leq t \leq \tau^* \\ f_{i,t}, \quad \text{for } t > \tau^*. \end{cases} \tag{1}$$

We will call a personal history $\overrightarrow{h}_{i,\tau^*}$ *consistent with* the protocol profile $\overrightarrow{f}$ if there is a non-zero probability that $\overrightarrow{h}_{i,\tau^*}$ will occur for player i under $\overrightarrow{f}$. If h_{i,τ^*} is consistent with $\overrightarrow{f}$ we call protocol $g_i(h_{i,\tau^*})$ *consistent* with $\overrightarrow{f}$, and when clear from the context we write g_i instead. Also, we denote the set of all g_i's, that is, all $g_i(h_{i,t})$'s for all $t \geq 1$, which are consistent with $\overrightarrow{f}$ by $\mathcal{G}_i^{\overrightarrow{f}}$.

Lemma 1 (Equilibrium characterization). *Consider a profile $\overrightarrow{f} = (f_1, f_2, \ldots f_n)$ of acknowledgement-based protocols. The following statements are equivalent:*
(i) $\overrightarrow{f}$ is an equilibrium.
(ii) $\forall$ player $i \in [n]$ $\begin{cases} \text{(a)} \quad C_i^{(\overrightarrow{f}_{-i}, g_i)}(\overrightarrow{h}_0) = C_i^{(\overrightarrow{f}_{-i}, r_i)}(\overrightarrow{h}_0) = C_i^{\overrightarrow{f}}(\overrightarrow{h}_0), \quad \forall g_i, r_i \in \mathcal{G}_i^{\overrightarrow{f}}, \text{ and} \\ \text{(b)} \quad C_i^{(\overrightarrow{f}_{-i}, g_i)}(\overrightarrow{h}_0) \leq C_i^{(\overrightarrow{f}_{-i}, r_i)}(\overrightarrow{h}_0), \quad \forall g_i \in \mathcal{G}_i^{\overrightarrow{f}}, r_i \notin \mathcal{G}_i^{\overrightarrow{f}}. \end{cases}$

Now we are ready to give anonymous, equilibrium protocols for $k = 2$ and $k = 3$. Let us define the following memoryless protocol with parameter $k \in \{2, 3\}$ which corresponds to the number of channels. Briefly, this protocol dictates to a player to split probability 1 equally on all channels, in every time-step.

Protocol $\mathbf{f^k}$: For any player i, every $t \geq 1$, and any transmission history,

$$f_{i,t}^k = \left(\Pr\{X_{i,t} = 0\} = 0, \quad \Pr\{X_{i,t} = a\} = \frac{1}{k}, \quad \forall a \in K \right). \tag{2}$$

n players - 2 transmission channels. By employing our characterization, we show that for $k = 2$ channels, f^2 is an equilibrium protocol for $n \in \{2, 3, 4\}$ players (Theorem 1). The next two lemmata are easily proved by Markov chain analysis which is omitted due to lack of space.

Lemma 2. *When all $n \geq 2$ players use protocol f^2 the expected latency of any player is $2^n/n$.*

Lemma 3. *For $n \geq 5$ players, f^2 is not an equilibrium protocol. In fact, a better response for any player is to not transmit in $t = 1$ and then follow f^2.*

Theorem 1. *For $n \in \{2, 3, 4\}$ players and $k = 2$ channels, f^2 is an equilibrium protocol with expected latencies 2, 8/3 and 4, respectively.*

Proof sketch. We show that the protocol profile where all n players use protocol f^2 is in equilibrium by showing that the condition (ii) of Lemma 1 holds. Starting with condition $(ii - a)$, assume a unilateral deviator i and an arbitrary protocol g_i consistent with $\overrightarrow{f}$. This protocol would dictate a history of transmissions h_{i,τ^*} with only "1" and "2" in it for some arbitrary $\tau^* \geq 1$, and then continue following f^2. The process of any such protocol, from the perspective of i is modelled as a Partially Observable Markov Decision Process, which due to the anonymity and uniformity of f^2, reduces to a Markov chain that yields expected latency $2^n/n$.

For condition $(ii-b)$, suppose i chooses a protocol r_i that is not consistent with $\overrightarrow{f}$. This means that there must exist some time $t < \infty$ for which $\Pr\{X_{i,t} = 0\} > 0$. Let us focus on the smallest such t, namely $t_0 \triangleq \inf\{t : \Pr\{X_{i,t} = 0\} > 0\}$. Now if we consider some arbitrary history $h_{i,t_0} = (a_{i,1}, a_{i,2}, \ldots, a_{i,t_0})$ and its respective protocol $r_i = r_i(h_{i,t_0})$ as in (1), one of two things can be true: either $a_{i,t_0} = 0$ or for $t > t_0$ protocol r_i is not identical to f^2. That is, we have the categories for r_i presented in Table 1. Note that the pairs of categories that r_i could be

Category 1	$a_{t_0} \neq 0$
Category 2	$a_{t_0} = 0$

Category I	$\forall t > t_0$: $\Pr\{X_{i,t} = 0\} = 0$
Category II	$\exists t > t_0$: $\Pr\{X_{i,t} = 0\} > 0$

Table 1. The categories of protocol $r_i(h_{i,t_0})$.

simultaneously are (1-I), (1-II), (2-I), and (2-II). By checking each of those cases and letting r_i be a best response, we show that no such protocol can yield expected latency to i lower than $2^n/n$. □

n players - 3 transmission channels. Similarly, in the case with $k = 3$ channels, we employ our equilibria characterization and show that f^3, defined in (2), is an equilibrium protocol for $n \in \{2, 3, 4, 5\}$ players. However, now we do not have an expression for the expected latency of a player such as the one of Lemma 2, thus, in order to follow the same method as before, for each $n \in \{2, 3, 4, 5\}$ under examination we have to find its expected latency individually.

Theorem 2. *For* $n \in \{2,3,4,5\}$ *players and* $k = 3$ *channels,* f^3 *is an equilibrium protocol with expected latencies* $3/2$*,* $15/8$*,* $189/80$ *and* $597/200$*, respectively.*

4 An Efficient Protocol with Expected Latency = ∞

In this section we give an anonymous, equilibrium protocol for the general case of $k \geq 1$ channels and any number of $n \geq 2k+1$ players. For this, we employ the deadline idea introduced in [4] and consequently used in [2,3]. Our protocol has the property that the time until all players transmit successfully is $\Theta(n/k)$ with high probability, even though the expected latency is infinite.

Consider $k \geq 1$ transmission channels, $n \geq 2k+1$ players, a fixed constant $\beta \in (0,1)$ and a deadline t_0 to be determined consequently. The $t_0 - 1$ time steps are partitioned into $r+1$ consecutive intervals $I_1, I_2, \ldots, I_{r+1}$ where r is the unique integer in $\left[-\log_\beta n/2 - 1, -\log_\beta n/2\right]$. For any $j \in \{1,2,\ldots,r+1\}$ define $n_j = \beta^j n/k$. For $j \in \{1,2,\ldots,r\}$ the length of interval I_j is $l_j = \lfloor \frac{e}{\beta} n_j \rfloor$. Interval I_{r+1} is special and has length $l_{r+1} = n/k$. We define the following protocol.

<u>**Protocol g**</u>: Every player among $1 \leq m \leq n$ pending players for $t \in I_j$ assigns transmission probability $1/\max\{n_j, k\}$ to each channel. Right before the deadline $t_0 = 1 + \sum_{j=1}^{r+1} l_j$ each pending player is assigned to a random channel equiprobably, and for $t \geq t_0$ always attempts transmission to that channel.

The proof of the following theorem is similar to that of Theorem 11 in [2] which considers the case with $k = 1$ channel, and is omitted due to lack of space.

Theorem 3. *Protocol* g *for* $n \geq 2k+1$ *players and* $k \geq 1$ *channels, is an equilibrium protocol and it is also efficient.*

References

1. Eitan A., Rachid E. A., and Tania J. Slotted aloha as a game with partial information. *Computer networks*, 2004.
2. G. Christodoulou, M. Gairing, S. E. Nikoletseas, C. Raptopoulos, and P. G. Spirakis. Strategic contention resolution with limited feedback. In *ESA*, 2016.
3. G. Christodoulou, M. Gairing, S. E. Nikoletseas, C. Raptopoulos, and P. G. Spirakis. A 3-player protocol preventing persistence in strategic contention with limited feedback. In *SAGT*, 2017.
4. A. Fiat, Y. Mansour, and U. Nadav. Efficient contention resolution protocols for selfish agents. In *SODA*, 2007.
5. A. B MacKenzie and S. B Wicker. Stability of multipacket slotted aloha with selfish users and perfect information. In *INFOCOM*, 2003.
6. J. Mo, H. W. So, and J. Walrand. Comparison of multichannel mac protocols. *IEEE Transactions on mobile computing*, 2008.
7. J. So and N. H Vaidya. Multi-channel mac for ad hoc networks: handling multi-channel hidden terminals using a single transceiver. In *Mobile ad hoc networking and computing*. ACM, 2004.